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Art. XI.—*The Design of an Induction Motor with large Air-Gap and Rotating Field Magnets.*

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(With 2 Text Figures,)

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It will easily be seen that the apparatus described in another paper[1] in these Proceedings is an induction motor, with the following peculiarities:—

1. The field magnets are magnetised by direct current, and are rotated to produce the rotating magnetic field. This is necessary in order that the axis of rotation of the magnetic field may be accurately determined, and that there may be no movement of the magnetic field along the axis of rotation.

2. The armature consists of a hollow cylinder of solid copper with a soft iron core.

3. The machine works at 100% slip, i.e., the armature remains at rest.

4. The air gap is very much larger than usual to permit of the insertion of the Dewar flask.

The problem of designing the instrument was similar to that of designing an induction motor. A first approximation to the behaviour of the instrument may be made by supposing the copper armature replaced by another armature of the same size, consisting of very narrow strips of conducting material insulated from each other, the resistance of all the strips in parallel being equal to that of the copper cylinder from end to end, and each strip being connected at either end with that diametrically opposite to it by a perfect conductor.

If this armature be placed in a uniform magnetic field of strength H, which rotates at the rate of N revolutions per second, the usual theory of the induction motor gives the torque as:—

$$\psi = \pi N H^2 l^2 c^2 R n (R^2 + 4\pi^2 N^2 L^2)^{-1}$$

where l cm. is the length of the armature, c.cm. is the diameter of armature. R. E.M. units is the resistance of each of the circuits (i.e., twice the resistance of one strip), n total number of

1. Laby and Roberts. A New Method of Determining the Mechanical Equivalent of Heat, page 148.

circuits (i.e., one-half the number of strips), and L.cm. the inductance of each of the circuits.

The quantity Hlc is the magnetic flux threading the armature, and is denoted by ϕ. Thus we have

$$\psi = \pi N n R \phi^2 (R^2 + 4\pi^2 N^2 L^2)^{-1}$$

We may write $R = \rho n$ where ρ is a constant depending on the resistance of the original copper cylinder.

Thus

$$\psi = \pi N \rho \phi^2 \left(\rho^2 + \frac{L^2}{n^2} 4\pi^2 N^2\right)^{-1}$$

Let $L = \lambda n$ where λ is a constant

$$\psi = \pi N \rho \phi^2 (\rho^2 + 4\pi^2 N^2 \lambda^2)^{-1} \text{........(1)}$$

It should be noticed that writing $L/n = \lambda$ (a constant) takes into account the mutual action of the induced currents on one another. For suppose that we replace the original copper armature first by one consisting of n circuits and second by one consisting of $2n$ circuits where n is large. The currents flowing in adjacent circuits will be nearly the same both in magnitude and in phase, and since the circuits are near together the mutual inductance of two adjacent circuits will be practically equal to the self-inductance of either of them. Thus the flux threading a circuit when a certain current flows in it will, in the case of the armature of $2n$ circuits, be approximately twice what it is for the same current in the armature of n circuits. Writing $L = n\lambda$ we assume that it is exactly twice the value.

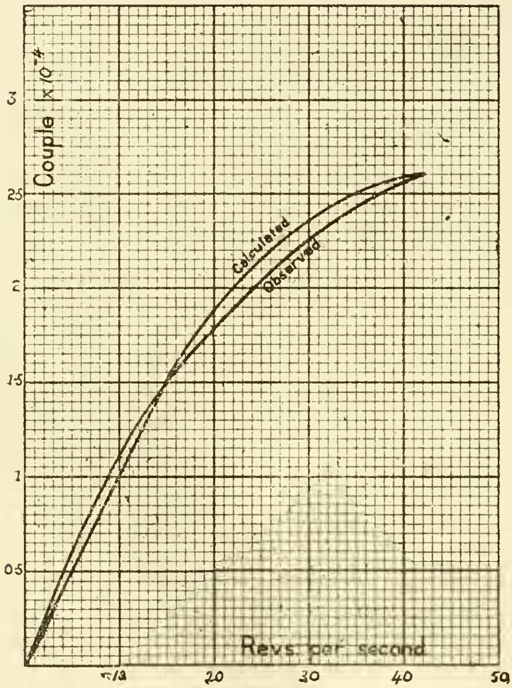
In order to determine ρ and λ for particular cases, the following experiments were carried out :—

Experiment (a).—A U-shaped permanent magnet was weighed and suspended by means of a bifilar suspension so that the poles hung downwards. An armature, consisting of a cylinder of copper with an iron core was placed midway between the poles and attached to a spindle, by means of which it could be rotated, a revolution counter was attached to the spindle. The couple acting when the spindle was rotated was measured by observing the deflection of a spot of light, which was reflected from a mirror attached to the magnet. The couple was measured for different rates of rotation of the armature. The value of ϕ was determined by winding the exploring coil of a Grassot fluxmeter around the armature and rotating it through 180°. This of course gives twice the value of $2.\phi$ The result obtained was $\phi = 6.23 \times 10^3/2$ Maxwell. Mass of magnet, 2870 gm. Distance from mirror to scale, 54.5 cm. Length of suspending wires, 44.2 cm. Distance between suspending wires, 2 cm.

The couples obtained were the following :—

Revolutions per second.	Deflection of light on reversing direction of rotation of copper.	Couple (absolute)
43.96	97.7 cm.	2.69×10^4
33.48	94.5 cm.	2.26×10^4
21.98	76.2 cm.	1.93×10^4
13.6	49.35 cm.	1.35×10^4

These results are plotted in Figure 1.



Using the values at two points we may determine ρ and λ from equation (1). From the graph when

$N=42$, $\psi=2{\cdot}6\times10^4$, and when $N=15$, $\psi=1{\cdot}5\times10^4$,

We have therefore

$$\left.\begin{array}{l}\rho=2{\cdot}772\times10^4 \text{ E.M.U.}\\ \lambda=\ 92{\cdot}5 \text{ cm.}\end{array}\right\}\cdots\cdots(2)$$

Substituting these values in equation (1) we calculate the couples ψ for different values of N, with the following results :—

$$N=20,\ \psi=1{\cdot}87\times10$$
$$N=30,\ \psi=2{\cdot}36\times10^4$$

These values are plotted in Figure 1, in the "calculated" curve.

Experiment (b).—Further experiments of the same nature were carried out by removing the armature from a series wound "Immisch" motor, and replacing it by a hollow copper cylinder with an iron core. The pole pieces were built up with cast iron so as to clear the copper by about one millimetre. The whole was mounted on a cradle dynamometer, and arranged so that the copper cylinder could be rotated at different speeds by means of a belt and pulleys of different sizes.

Current was passed through the field coils and the torque produced on rotating the copper measured in the usual way.

The value of ϕ was measured by making and breaking the current in the field coils, and noting the deflection of a fluxmeter with an exploring coil, wound around the copper. In each case the couple with zero flux was measured to eliminate the frictional couple. The following results were obtained :—

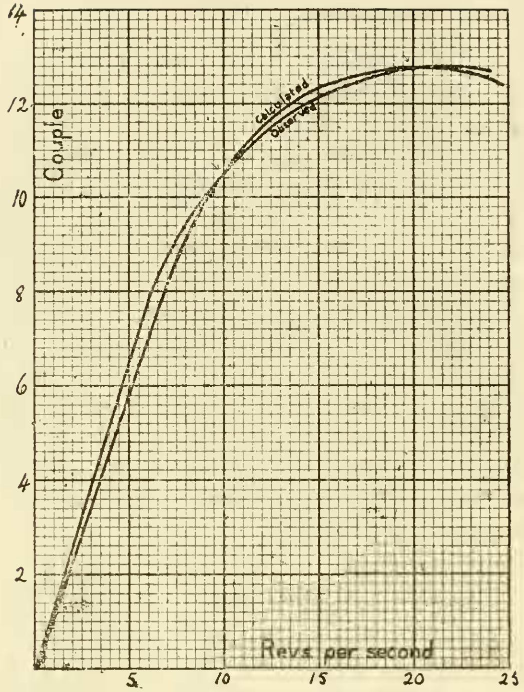
Revolution per second.	Value of 2ϕ	Mass of balancing rider.	Distance of rider from Centre.	Couple due to Eddy Currents.	Couple corrected to $\phi = 2{\cdot}79 \times 10^5/2$
6.65	0 2.72×10^6	100 200	29 cm. 52.5 cm.	7.45×10^6	7.84×10^6
11.0	0 2.74×10^5	100 200	10 cm. 60 cm.	1.08×10^7	1.12×10^7
16.1	0 2.79×10^5	100 200	14 cm. 70 cm.	1.23×10^7	1.23×10^7
20.2	0 2.79×10^5	100 200	16 cm. 73 cm.	1.27×10^7	1.27×10^7
24.0	0 2.79×10^5	100 200	10 cm. 70 cm.	1.27×10^7	1.27×10^7

In order to correct the observed couples for variations in the value of ϕ it was assumed that the couple is proportioned to ϕ^2. To verify this, two experiments were carried out at the same rate of rotation with values of ϕ in the ratio of 1 to 2. The couples were measured as before. The values of the ratio Couple: ϕ^2 were found to be

$$6.07 \times 10^3 \text{ and } 6.04 \times 10^3.$$

This justifies the assumption made.

The values of the couples corrected in this way are given in the last column of the table, and are plotted against revolutions per second in Fig. 2.



In order to compute the values of ρ and λ of Equation (1) for this case two points on the graph are used.

When N=10, $\psi=10.4\times10^6$, and when N=20, $\psi=12.7\times10^6$

In the same way as before we get

$$\left.\begin{array}{l}\rho=4.63\times10^4\text{ E.M.U.}\\ \lambda=382\text{ cm.}\end{array}\right\}\ldots\ldots(3)$$

Using these values of λ and ρ the couples which should correspond to different values of N can be calculated from formula (1) to be :—

$$N=15,\ \psi=12{\cdot}33\times10^6 \text{ and } N=25,\ \psi=12{\cdot}29\times10^6$$

These are plotted in Fig. 2 in the calculated curve.

In order to apply these results to the design of a new apparatus, it is necessary to compare the values of ρ as calculated above with the values of the resistances from end to end of the two cylinders which were used. If we do this we get the following :—

ρ/Resistance of copper cylinder =(a) 17·6 and (b) 24·2......(4)

The values of λ obtained were λ=(a) 92·5 and (b) 382......(5)

The reason for the fact that ρ is about twenty times the resistance of the copper from end to end, which we may call σ, is that if r is the resistance of each of the $2.n$ strips by which the copper is replaced,

$$R=2n\sigma$$

The resistance R of each of the circuits formed by joining a pair of strips in series by two perfect conductors will be

$$R = 2r = 4n\sigma$$

$$\text{But } R = n\rho \text{ and } \therefore \rho = 4\sigma$$

That is, if the ends of the conducting strips were joined by perfect conductors, we would have ρ equal to four times the resistance of the copper from end to end. The remaining factor of five is due to the fact that the effective resistance of the paths joining the strips is not zero.

The large increase in the value of λ in the experiments with the Cradle dynamometer is probably due to the fact that in these experiments the air gap between the copper and the pole pieces was smaller than in the case of the permanent magnet.

Another point, which must be determined, before it is possible to design a new apparatus, is the extent to which the magnetic resistance of a magnetic circuit can be inferred from the dimensions of the apparatus.

To do this, the "Immisch" motor was used, and the flux threading the iron core was measured for different diameters of the core, and different magneto-motive forces, with the following results :—

Average area of air gap.	Length of air gap (both sides).	Magnetic resistance of air gap.	Magneto Motive force.	Magneto Motive force. Resultant Flux.
60.43	0.73	0.0121	3.47×10^3	0.0251
,,	,,	,,	6.01×10^3	0.0301
39.4	1.03	0.0261	3.70×10^3	0.0380
,,	,,	,,	6.61×10^3	0.0418
28.8	1.87	0.0650	3.71×10^3	0.0695
,,	,,	,,	6.84×10^3	0.0630

The numbers in the last column give the effective magnetic resistance of the Circuit.

Comparing these values with the values of the resistance as calculated from the dimensions, it will be noticed that for small air gaps, the effective resistance is larger than the calculated, and also that the value increases with increasing flux density. This means a higher magnetic leakage at higher flux densities. But when the air gap is 2.cm. long, the calculated and effective magnetic resistances are practically the same, and also the value of the effective resistance is almost independent of the flux density.

Certain dimensions in the new apparatus are fixed :—

(i.) The external diameter of the rotor is limited to 30 cm.

(ii.) The internal diameter of the pole pieces must be not less than 7.3 cm.

(iii.) The external diameter of the armature must not exceed 5.5 cm.

(iv.) The length of the armature must not exceed 13 cm.

(v.) Rough calculations shewed that the thickness of the copper should be 3 or 4 mm.

The copper was made 3.3 mm. thick, and 5.45 cm. in external diameter.

This makes the electrical resistance from end to end 4.46×10^3 E.M. unit.

From equation (4) we multiply this by 20 to obtain the value of

$$\rho = 20 \times 4.46 \times 10^3$$

Calculating the magnetic resistance from the dimensions we obtain the value

$$\text{Magnetic resistance} = 0.040.$$

Since the air gap is 1.9 cm. long it can be inferred from the experiments carried out that this will be the effective value of the magnetic resistance.

The flux obtainable is therefore given by

$$\phi = .4\pi(ni)/0.04 = 10\pi(ni)$$

where ni is the number of ampere turns.

It is proposed to run the apparatus at 1500 revolutions per minute, i.e., $N = 25$.

$$\begin{aligned}\text{Power} &= 2\pi N\psi \\ &= 2\pi^2 N^2 \rho\phi^2(\rho^2 + 4\pi^2 N^2 \lambda^2)^{-1}\end{aligned}$$

Since $\frac{1}{2}$-Horse Power is required we have

$$3.73 \times 10^9 = 1{\cdot}086 \times 10^{12}(ni)^2(7.95 \times 10^9 + 2.47 \times 10^4\lambda^2)^{-1} \ldots\ldots (6)$$

If $\lambda = 92$ and 382 as in Equation (5) we obtain respectively

$$ni = 5.38 \times 10^3 \text{ and } 5.86 \times 10^3 \ldots\ldots (7)$$

The inductance term should not be greater than 356, as the air gap is nearly ten times as large as in the Experiment (b).

The heat generated in a winding of given size depends on the number of ampere turns.

It is, therefore, necessary to discover whether it is possible to dissipate the energy which would be generated in a winding of the size required by the dimensions of the apparatus. In making calculations it can be safely assumed that .05 watt can be dissipated per square cm. of the area of winding. With this assumption the possible number of ampere turns on each coil comes out to be 4500—that is, a total of 9000, which is more than is required. As only a limited voltage is available it is necessary to choose wire of such a gauge that the resistance will be low enough to allow the requisite number of ampere turns.

The poles were wound with 4720 turns of 24 gauge copper wire.

An experiment was carried out to determine the power of the apparatus so designed. It was found that when the apparatus was generating one half horse-power the current in the field winding was .575 amperes; since there are altogether 9440 turns this gives for magnetising current,

$$5.43 \times 10^3 \text{ ampere turns.}$$

This value lies between the predicted limits, viz.,

$$5.38 \times 10^3 \text{ and } 5.86 \times 10^3 \text{ as in equation (7).}$$

The rise of temperature of the winding was found from its resistance, the temperature coefficient of a sample of the wire having been measured. The temperature rose 47° with all the vents in the apparatus closed. Thus the apparatus realised the power for which it was designed. The ventilation caused by the vents was large, and with them open the rise was much smaller.